On the ’Wing... #164

Twist Distributions for Swept Wings, Part 4

This installment will be devoted to just two items: the interrelationship of lift distribution, aileron configuration, and adverse and proverse yaw, and how the bell-shaped lift distribution can be utilized to reduce induced drag.

Defining lift distributions

Before describing research results related to lift distributions, another look at the elliptical lift distribution is in order.

In Part 1, the elliptical lift distribution was defined by means of a geometric construction. Figure 1 illustrates this methodology. Simply stated, vertical lines are dropped from a semicircle to the baseline. The center of these verticals are then determined and a curve drawn which connects the determined points. The curve thus defined is an ellipse. This shape is then used as a basis for the lift distribution across the span. The result of such a lift distribution is a constant downwash across the entire span and a minimization of induced drag.

As an extension of the description in Part 1, there is another method of defining the elliptical lift distribution which involves trigonometric functions. In this construction, the point P is defined by its X and Y coordinates as determined by the following formulae:

$$YP = b/2 * \cos \xi$$
$$XP = K * \sin \xi$$

For the construction of the semicircle, K = b/2, the semi-span. For the construction of an ellipse, K can be any value less than one. In the illustrated case, Figure 2, K = 1/2 in keeping with the geometric construction explained previously.

It should be noted at this point that each point P' defines the lift generated by that wing section, the coefficient of lift times the local chord. One way of visualizing this is to consider an elliptical lift distribution and an elliptical wing operating at a coefficient of lift of one. Remember, the lift coefficient is constant across the span; that is, the local coefficient of lift for each wing segment will be one. In this case, the wing chord is directly proportional to the height of the lift distribution curve at that point along the Y-axis.

Taking this trigonometric methodology one step further, we can modify the trigonometric function by adding an exponent n. For example, rather than using $\sin \xi$, we use $\sin^n \xi$. See the included Table for an idea as to how various exponents affect the resulting points P'.

Figure 3 shows the elliptical lift distribution, $\sin \xi$, and three other distributions, $\sin^{2.5} \xi$, $\sin^3 \xi$, and $\sin^4 \xi$. Because the aircraft weight is held constant, the area under each curve is identical. The latter lift distributions which utilize the ‘n’ exponent are termed bell-shaped for obvious reasons.

When the bell-shaped distribution is applied to moderately swept back wings, the following generalizations apply: When the exponent n is two, the lift distribution is bell-shaped but there is no induced thrust at the wing tips. When n = 2.5, the adverse yaw disappears and proverse yaw begins to appear. As n approaches three, the induced drag begins to increase rapidly. The designer should therefore use the lowest value of n in keeping with his/her objectives. The Hortens used n = 3 for most of their designs, but n = 2.5 may be sufficient for use on models where both adverse and proverse yaw are undesirable and induced drag should be as low as possible.

Yaw moment, lift distribution, and aileron configuration

Dr. Edward Udens analyzed the yawing moment of two swept wing planforms with differing lift distributions and control surface configurations. Figure 4 shows the various configurations, notes their lift distributions, and presents the yaw moment for each. The elliptical and $\sin^3 \xi$ bell-shaped lift distributions were evaluated. Negative yaw moment values indicate adverse yaw, positive yaw moment values indicate proverse yaw.

Both of the wings with elliptical lift distributions demonstrate adverse yaw regardless of control surface placement. Proverse yaw can be generated by using the bell-shaped lift distribution and by keeping the elevon control surface well outboard.

Dr. Udens' results demonstrate an increasing adverse yaw moment as the elevon control surface is moved inboard. This is an important consideration. The roll control surfaces must be placed in the area of the wing which has a concave lift distribution curve; that is, outboard in the case of the bell-shaped lift distribution. Although the Hortens used the $\sin^3$ lift distribution, they included inboard elevons which may have significantly reduced the proverse yaw moment and in fact created an adverse yaw moment.

A relevant example

There are a number of readers who at this point desire some sort of practical example of the bell-shaped lift distribution generating proverse yaw as elevon control surfaces induce a roll moment. Ideally, we would look for a swept wing tailless model without winglets which exhibits very strong adverse yaw as an example. Those who have built and flown a Klingberg wing know well this model meets the ideal. Don Stackhouse of DJ Aerotech had the following to say about his Klingberg wing:

> "My stock Klingberg, with its horrible adverse yaw and a yaw-roll coupling that essentially negates the roll response to any but the smallest elevon deflections, is essentially unsafe to fly in any place with maneuvering space restrictions or in any kind of turbulence."

Don goes on to say that the addition of any aileron differential severely affects the aircraft in pitch. The application of down elevator to inhibit the nose up pitching reduces the differential, so it's a Catch 22 situation. Don has not flown his Klingberg wing in several years, and in fact only takes it out of storage to serve as an exhibit model.

Michael Allen, a student at Embry-Riddle Aeronautical University and an intern at NASA Dryden Flight Research Center under Al Bowers, decided to build a Klingberg wing using a bell-shaped lift distribution. The taper ratio and other planform parameters of the two meter Klingberg wing closely match those of the Horten Xc, an advanced ultra-light glider designed by Reimar Horten while he was living in Argentina. Al was able to get the twist values for the modified Klingberg wing from Reinhold Stadler, and Michael built the model using the defined twist distribution.

Additionally, Michael used an elevon planform, illustrated in Figure 5, in keeping with the results of Dr. Udens (Figure 4 No. 6). This elevon planform is calculated to give a small amount of proverse yaw, $Cn\partial a = 0.001942$.

Al says the wing looks very "organic" in the air, and while flying directly overhead and giving full right or left stick, there is not even a hint of adverse yaw in evidence.

Reducing induced drag

"The elliptical lift distribution is the most efficient." We have heard this statement often over the years. Recently we've come to discover it is not entirely true simply because it is incomplete. More accurately, "the elliptical lift distribution is the most efficient for a wing of given lift and span." The qualifications may not seem to be of much importance at first. But consider a wing of a given span with an elliptical lift distribution. Is there a way to reduce the induced drag of this wing, making it more efficient, while keeping the root bending moment the same?

If you simply add span and maintain an elliptical lift distribution, the wing will be more efficient because you've increased the aspect ratio. But the spar will need to be strengthened because the bending moment at the root will have been increased with the larger span. So the question becomes a matter of finding a means to increase the span without increasing the load at the wing root. Enter the bell-shaped lift distribution.

Ludwig Prandtl came up with the elliptical span load around 1908, but did not formally publish his work until 1918. In 1933, Prandtl published his paper "On the minimum induced drag of wings" in which he presented the bell-shaped lift distribution. Prandtl's solution provided an 11% reduction in induced drag with a 22% increase in span and no increase in the root bending moment. In 1950, Robert T. Jones looked at the same problem and, unaware of Prandtl's work, came up with a similar solution by a different means.

Jones' computations show a 15% decrease in induced drag with a 15% increase in span when using a bell-shaped span load. Figure 6 illustrates Jones' planform, a comparison to the standard elliptical lift distribution, and the trapezoidal shape of the produced downwash.

Also included in that illustration is a diagram showing the lift distribution for a wing with a span ratio of 1.30 and a root bending moment identical to the span ratio 1.0 elliptical wing. Jones states that while the span can be increased further, the near maximum benefit comes with a 15% increase in span.

Other investigators, notably Klein and Viswanathan, have looked at the same constant root bending moment problem but also included other constraints, such as shear. The results point to a bell-shaped lift distribution and similar reductions in induced drag.

Back to winglets

In Part 3, we described how winglets can be a source of induced thrust. We also drew a parallel between the action of winglets and the effects of generated upwash on the outer portion of a swept wing. Consider a wing with a bell-shaped lift distribution which is producing induced thrust at the wing tips to be equivalent to a wing with winglets which is operating at its design speed.

While researching this series of articles, we ran into a document produced by Boeing as part of their publication *Aero* dealing with blended winglet design for various passenger and cargo aircraft. Briefly, the addition of properly designed winglets which extend the wing between ten and 16 percent can substantially increase payload and range and decrease takeoff runs, particularly near maximum gross weight. This is parallel to the effects predicted for the span extension proposed by Jones.

According to the article, maximum payload increases, takeoff runs are shortened, cruise drag is decreased by four to more than five percent, and range is increased by approximately four percent. This is evidence that, when properly designed, winglets can improve performance over a wide speed range. Additionally, blended winglets improve directional and pitch stability and longitudinal and lateral trim stability. There is no change in stall speed or Dutch roll damping.

One of the interesting points covered in the article involved the toe angle of the winglet. Initially, the toe out angle was set for zero degrees. While this minimized induced drag, it imposed very high loads on the wing. A toe out angle of two degrees reduced the bending loads on the wing but did not adversely affect the drag reduction except in the flaps down position. Boeing determined this was an acceptable trade-off for reducing required structural modifications.

It’s important to realize that commercial aircraft have span limitations based on constraints imposed by airport architecture, so vertical winglets are a much more attractive option than increasing the wing span. Boeing’s blended winglets aerodynamically increase the wing span without imposing a greater root bending moment and without increasing the actual wing span.

Discussion

The following discussion recently took place on the nurflugel e-mail list. We think the exchange may be enlightening, particularly for those readers with some doubts as to the efficacy of the bell-shaped lift distribution as applied to reducing induced drag. Al Bowers is Chief of Aerodynamics at NASA Dryden Flight Research Center.

From: Al Bowers <al.bowers@dfrc.nasa.gov>
Date: Wed, 18 Jun 2003 07:58:24 -0700
Subject: [nurflugel] NASM and Hortens...

Just a quick FYI: Russ Lee is in a blurb about the Hortens at:
<http://www.airandspacemagazine.com/ASM/Mag/inthemuseum.html>

The blurb is mostly right. The part about a drag penalty isn't quite true (but I made that mistake in the past, so I can't complain too much). Nice photos...

Al

From: Russell Lee <russlee_99@yahoo.com>
Date: Tue, 24 Jun 2003 10:37:51 -0700 (PDT)
Subject: Re: [nurflugel] Digest Number 1143

Al, when the author of the A & S piece asked me about Horten's bell distribution, I recalled that you had reported finding less drag than with the elliptical distribution. I wanted to mention that fact but I have no idea how this occurs, so wanting to err on the cautious side, I recited the standard litany about the drag penalty with bell.

Would you have time to explain why the bell drag is less? I would sure like to give Reimar full credit when people ask about his work.

Russ Lee

From: Al Bowers <al.bowers@dfrc.nasa.gov>
Date: Wed, 25 Jun 2003 15:33:46 -0700
Subject: Re: [nurflugel] Digest Number 1143

Hey Russ,

The question is actually pretty complex. But the problem boils down to one issue: is span constricted or not? If span is constricted, then the lowest drag is elliptical (unless winglets are allowed). If span is not constricted, then the bell shaped is lower drag.

Let's assume we design a wing elliptical. Now, given that wing, what is the size of the spar we have to build to support that load (this is the wing root bending moment, hereafter the WRBM). Now, as a thought experiment, ask the question:

Is there a span and a span load that results in SAME WRBM but has less drag? If the answer is yes, then what is the optimum span and span load for the same WRBM? (This was Prandtl's question in 1933.)

The answer is yes, it is the bell shaped load distribution. The BSLD has the same WRBM as the elliptical and the same lift (so the same wing spar), but it has 22% MORE span and 11% LESS drag. It's the optimum drag for a given wing spar (which makes it of interest to birds; why haul around more bio-mass than necessary?).

By the way, this is also the subject of R.T. Jones 1950 paper, as well as being developed a bit more with Klein and Viswanathan's 1975 paper.

The other piece of the puzzle is the induced thrust at the wing tips (ala winglets). This allows the defeat of the adverse yaw part. But that's another story.

Does that help?

Al

From: "DavidRSw" <DavidRSw@bdumail.com>
Date: Wed, 25 Jun 2003 16:57:18 -0700
Subject: Re: [nurflugel] ESLD vs. BSLD

Hi Al,

So after listening to you give at least three lectures on this subject, talking to you in person an equal number of times, and reading your posts here, finally I may be beginning to understand what you are saying. ;-)

So, if we take two aircraft with the same weight = same lift = same WRBM = same wing spar, then we have two aircraft with different spans and different drags?

One hang glider with an elliptical lift distribution (ESLD) wing that weighs 245 lbs. ready to fly, has a 40 ft. span and a glide of 22:1. Does the other hang glider with the bell shaped lift distribution (BSLD), weigh 245 lbs. ready to fly, have a 48.8 ft. span, and a glide of 24:1? Have I got it right yet?

Disregarding the weight, what would the glide be of a 48.8 ft. span hang glider with an ESLD? If we have two hang gliders of 48.8 ft. span, the one with the ESLD will have a better glide but weigh more? About how much more?

Thanks,
Dave Swanson
Glendale, CA

From: "Albert Robinson" <arobins1@midsouth.rr.com>
Date: Thu, 26 Jun 2003 02:34:47 -0500
Subject: Re: [nurflugel] ESLD vs. BSLD

Or with the BSLD are we are driven to a longer span to make up for the losses incurred? How else would you have the same WRBM with a greater span? I thought one premise of BSLD was an increase in stability but with a small loss in total efficiency. Would not ESLD for a given span have to be more efficient? For that matter, were any of the Horton designs ever tested and demonstrated as "pure" BSLD? Or perhaps a blend of both.

Sorry, just old and stupid, I don't understand.

A-n-P

From: Al Bowers <al.bowers@dfrc.nasa.gov>
Date: Thu, 26 Jun 2003 08:28:46 -0700
Subject: Re: [nurflugel] ESLD vs. BSLD

>So, if we take two aircraft with the same weight = same lift = same WRBM =
>same wing spar, then we have two aircraft with different spans and different
>drags?

RIGHT!

>One hang glider with an elliptical lift distribution (ESLD) wing that weighs
<snipped>
>ready to fly, have a 48.8 ft. span, and a glide of 24:1?

Close, it's only the INDUCED drag we reduced a bit (~11%), we didn't do anything to the profile drag (remember, we changed the wing area, so the balance of drag would be different, but profile drag changes would be "insignificant"). So the "real" L/D max would probably only go up to a little over 23:1 (not quite to 24:1). And I would imagine the weight would rise a little as well (we did not consider the shear required to carry that load further out, Prandtl's original solution didn't consider shear, but Klein's & Viswanathan's solution DID).

> Or with the BSLD are we are driven to a longer span to make up for the
<snipped>
> and demonstrated as "pure" BSLD? Or perhaps a blend of both.

BSLD gets less drag than ESLD, that's not a "making up for losses" in my mind. The reason BSLD has the same Wing Root Bending Moment (WRBM) as ESLD is because BSLD carries less load out near the tips as ESLD.

It's like trying to pick up a 40 lb. tool box and turn it around vs. picking up a 40 lb. ladder and turning it around. The ladder has more mass out further away than the tool box, so you have to apply more load to turn it. BSLD is the tool box and ESLD is the ladder.

There is no loss in efficiency, BSLD is BETTER than ELSD. It minimizes the structure to carry the load (or you carry more payload as a fraction of total weight), it gets less drag, and it also solves the adverse yaw problem (you don't NEED a vertical tail, for even less weight).

It is the bird flight solution.

> Sorry, just old and stupid, I don't understand.

No, I’m just not explaining it well enough. When you get this one, a HUGE light bulb will turn on in your head and you’ll suddenly get it, in a BIG way! I get chills just thinking about this, it is so completely right, elegant, and simple.

It HAS to be the bird flight solution.

From: Al Bowers <al.bowers@dfrc.nasa.gov>
Sent: Thursday, June 26, 2003 10:28 AM
Subject: Re: [nurflugel] ESLD vs. BSLD

OK, OK, I think I am getting it now, but one more pass for us “aerodynamically impaired”:

So to get simplistic (it’s more than this, I am sure) the BSLD it is a function of airfoil selection and of course washout (type and style of twist). You said that the WRBM would be less with BSLD because the tips are carrying less load than if it were ESLD. (Am I doing OK so far?) In order for it to be “less” something had to go away, i.e.: total lift in the wing tip sections correct? But the trade off is worth the loss in that the lift vector at the tips is now forward (from the washout and the airfoil selection) and provides yaw stability plus less bending moment. Hooowee, do I got it?? :) or maybe a little??

From: Al Bowers <al.bowers@dfrc.nasa.gov>
Date: Thu, 26 Jun 2003 10:07:45 -0700
Subject: Re: [nurflugel] ESLD vs. BSLD
Albert Robinson writes:

> OK, OK, I think I am getting it now, but one more pass for us
<snipped>
> I got it?? :) or maybe a little??

BSLD is a function of twist, design point (lift coefficient & wing sweep), and airfoil. But ESLD is also a function of airfoil selection as well. And we traded the lift at the wing tips and moved it inboard a bit relative to ESLD.

	ESLD	BSLD
center load:	less	more
tip load:	more	less
total load:	equal for both	

I think you’ve got it now...

Al

The bell-shaped lift distribution and tailless RC sailplanes

For AMA RC models outside of the Unlimited class (RC-HLG, 2M, Standard), span is limited. Designing a tailless model with a bell-shaped lift distribution in an attempt to improve performance beyond that of a conventional tailed aircraft of the same span is therefore problematic, as Al Bowers explains.

Still, for the Unlimited class, where the only limitations are wing area (2325 sq. in.), mass (5 Kg., 11.02 lbs.), and wing loading (3.95 - 24.57 oz./sq.ft.), a competitive swept wing tailless model is certainly in the realm of possibility, and in fact, may be the best choice.

A tailless model utilizing the bell-shaped lift distribution is a particularly enticing proposition when such considerations as ground handling and construction costs are removed and modern low Reynolds airfoils, vortex-lattice computer codes, and high-tech materials and fabrication methods can be so easily added to the design and construction processes.

Our sincere appreciation goes to Al Bowers for providing substantial guidance and positive reinforcement, as well as a number of printed references, for this installment. Thanks are also due to the members of the nurflugel e-mail list for their informed questions regarding the elliptical and bell-shaped lift distributions.

The next and final installment in this series will provide a summation of the Horten, Culver, and Panknin twist distribution methodologies.

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Ideas for future columns are always welcome. *RCSD* readers can contact us by mail at P.O. Box 975, Olalla WA 98359-0975, or by e-mail at <bsquared@appleisp.net>.

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Table 1: Sin^n values for the construction of lift distributions

ξ	90°	78.75°	67.5°	56.25°	45°	33.75°	22.5°	11.25°
$\sin \xi$	1.0000	0.9808	0.9239	0.8315	0.7071	0.5556	0.3827	0.1951
$\sin^2 \xi$	1.0000	0.9619	0.8536	0.6913	0.5000	0.3087	0.1465	0.0381
$\sin^{2.5} \xi$	1.0000	0.9526	0.8204	0.6304	0.4204	0.2301	0.0906	0.0168
$\sin^3 \xi$	1.0000	0.9435	0.7886	0.5748	0.3536	0.1715	0.0560	0.0074
$\sin^4 \xi$	1.0000	0.9253	0.7285	0.4780	0.2500	0.0953	0.0214	0.0014

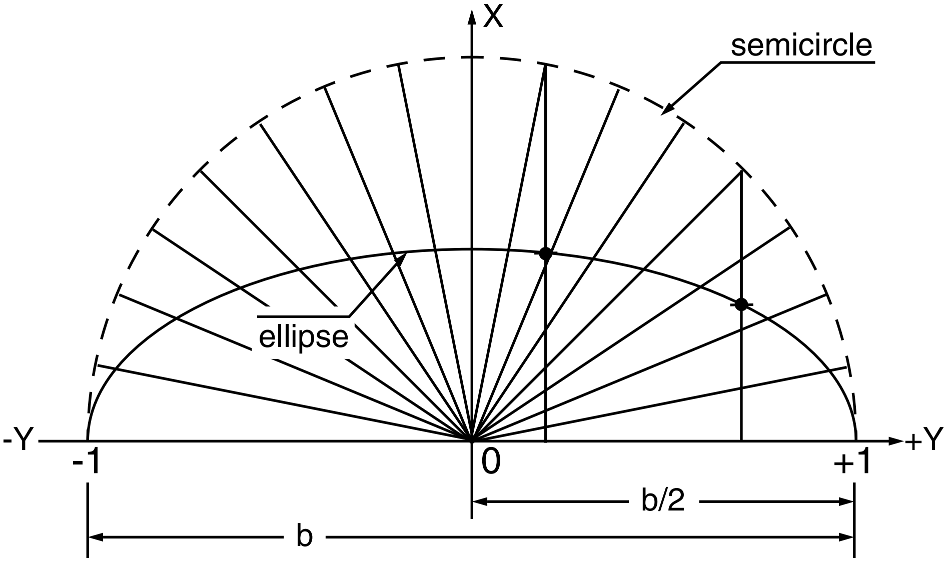


Figure 1. Geometric construction of elliptical lift distribution

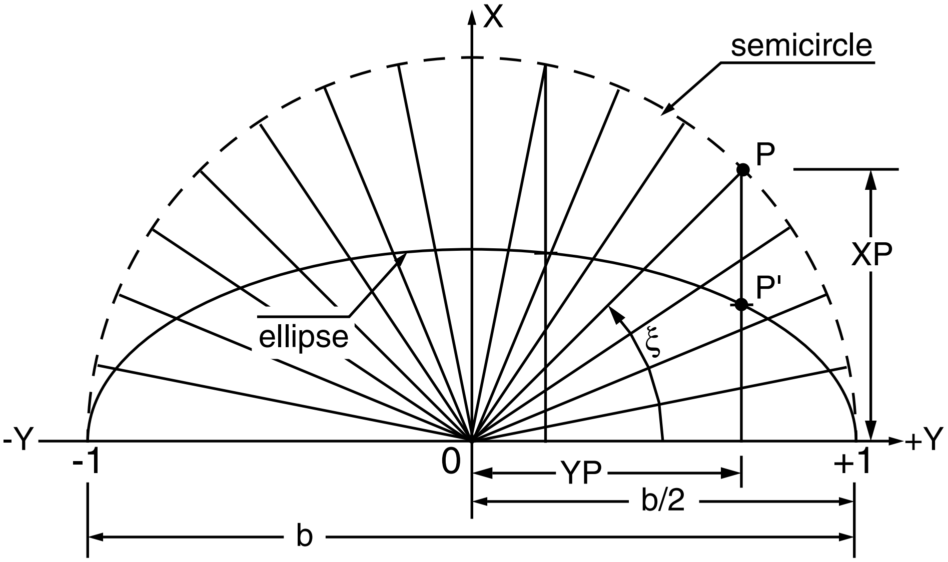


Figure 2. Trigonometric construction of elliptical lift distribution

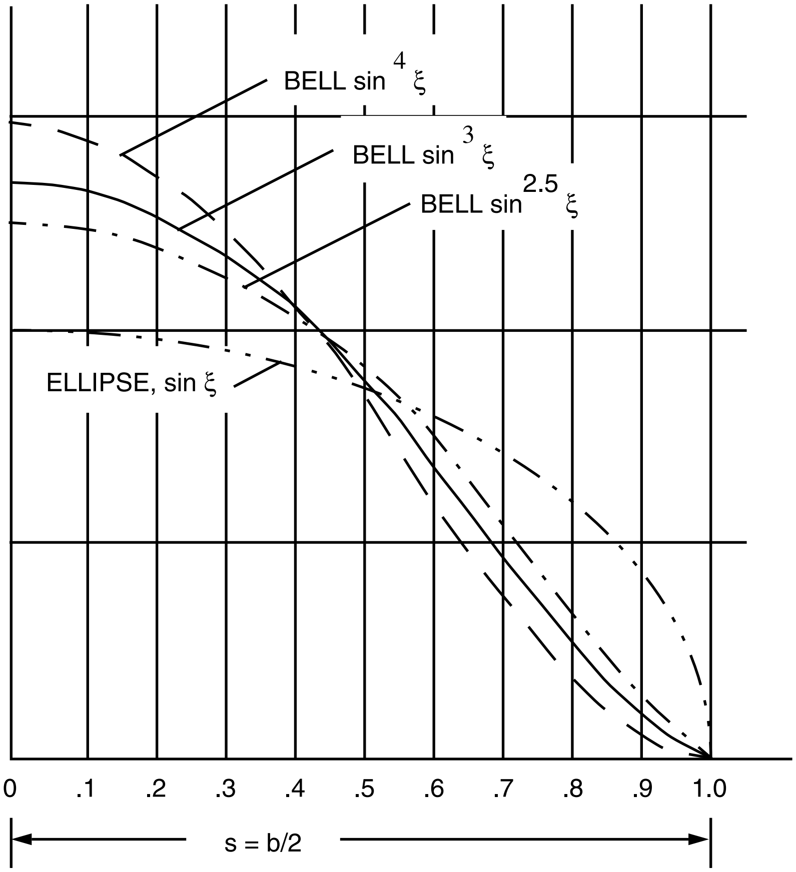
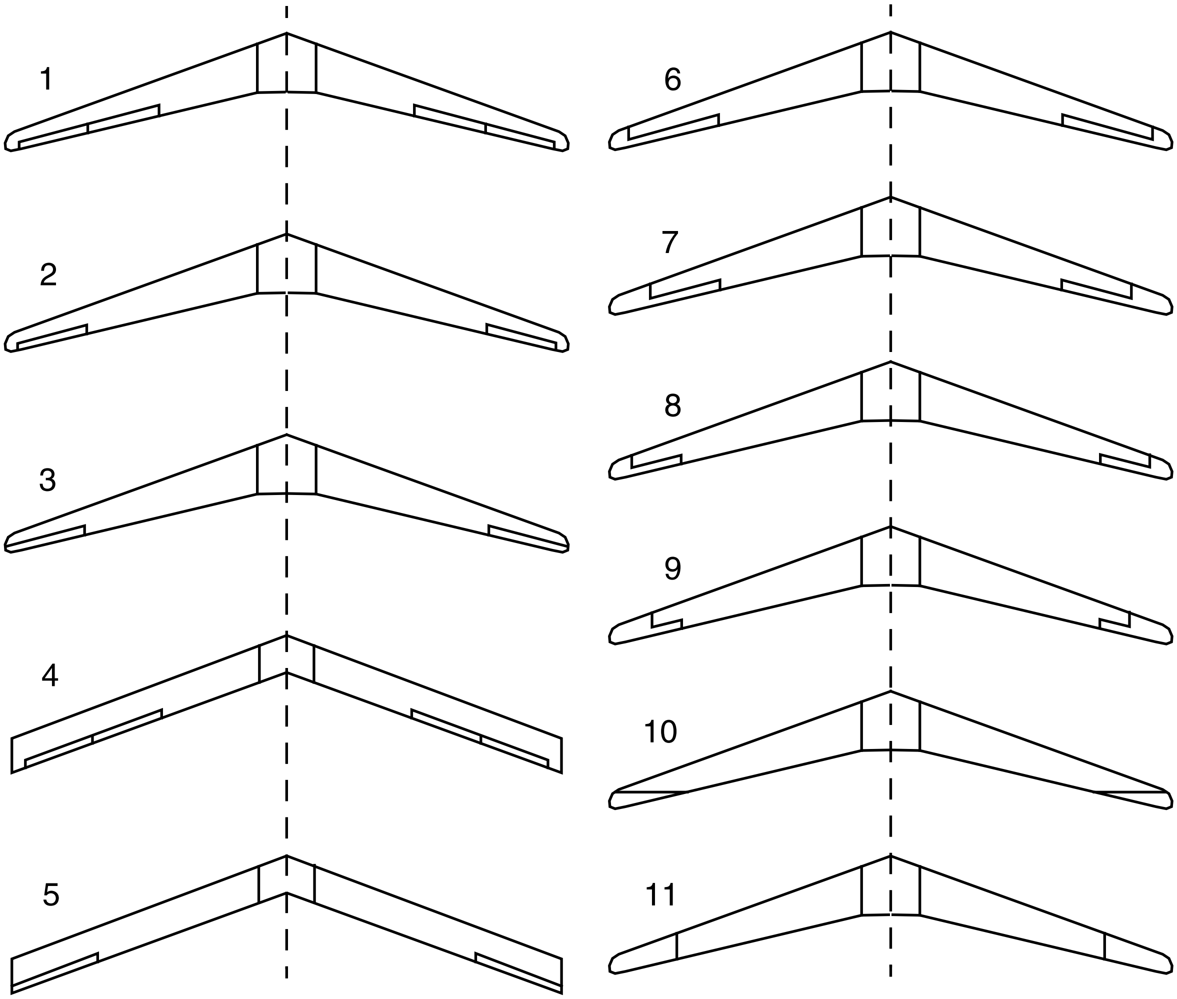


Figure 3. Lift distributions for $\sin^n$ functions.



Elevon Configuration	Cn∂a	Span Load
1	-0.002070	bell
2	0.001556	bell
3	0.002788	bell
4	-0.019060	elliptic
5	-0.015730	elliptic
6	0.001942	bell
7	0.002823	bell
8	0.004529	bell
9	0.005408	bell
10	0.004132	bell
11	0.005455	bell

Figure 4. Relationship of control surface configuration, span load, and yaw moment.

Photo 1

Michael Allen and his “Hortenized” Klingberg ’wing.

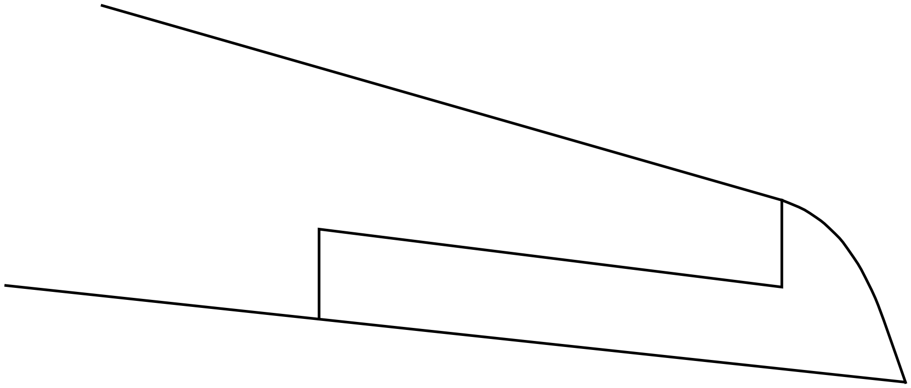
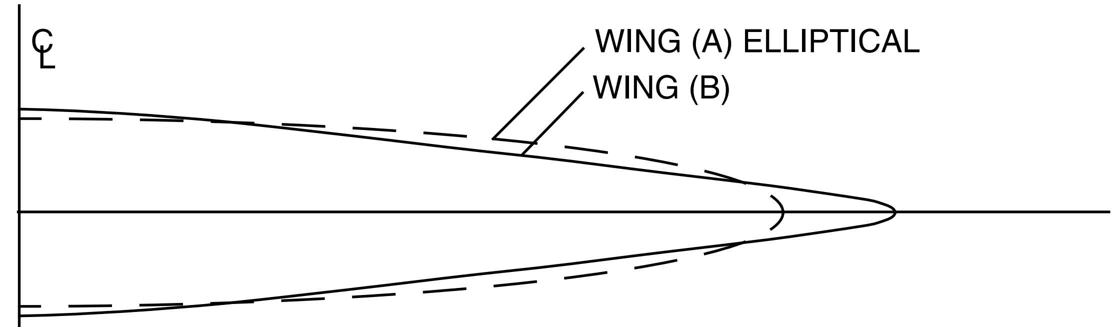
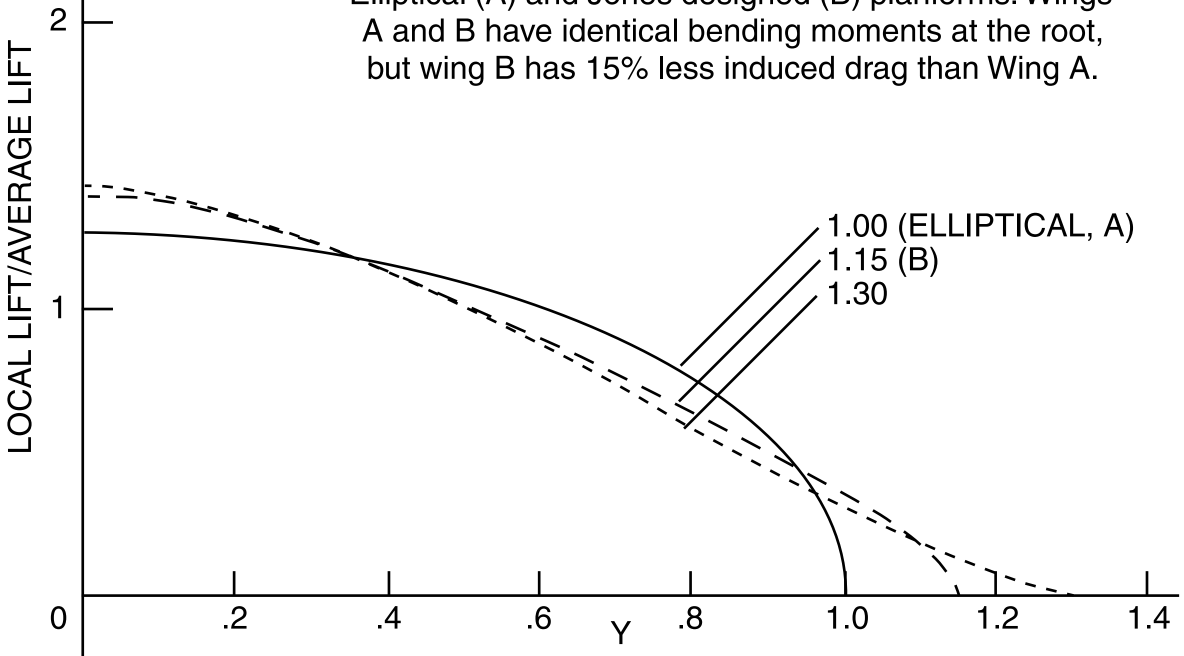


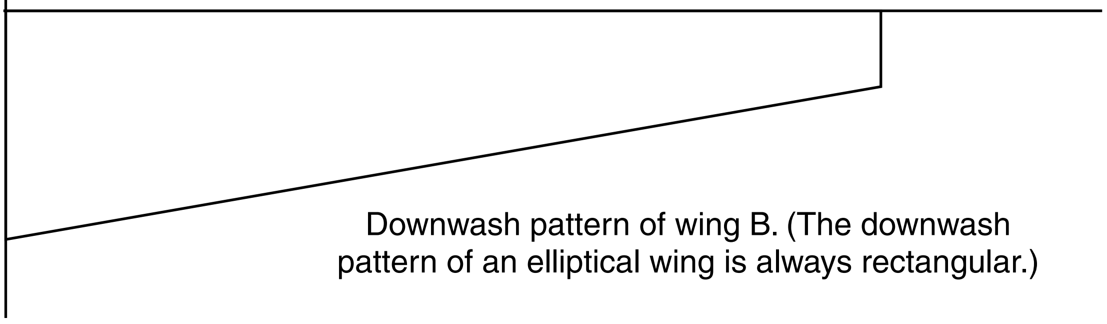
Figure 5. Elevon planform for Michael Allen’s “Hortenized” Klingberg ’wing.



Elliptical (A) and Jones designed (B) planforms. Wings A and B have identical bending moments at the root, but wing B has 15% less induced drag than Wing A.



Lift distributions for elliptical wing and wings with span ratios of 1.15 (B) and 1.30., root bending moment constant. As span ratio increases, tendency is toward bell-shaped distribution.



Downwash pattern of wing B. (The downwash pattern of an elliptical wing is always rectangular.)

Figure 6. Comparison of various aspects of Jones' low induced drag planform to an elliptical wing.